MATEMÁTICAS CCSS II
ÁLGEBRA **JUNIO 2019 A**
PROBLEMA 37

Problema 1. Dadas las matrices

$$A=\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{y} \quad B=\begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix}$$

Se pide:

a) Calcular $(AB)^{-1}$.
b) Calcular $AB^t - A^tB$.
c) Resolver la ecuación $B^tX + A^tB = A^t$.

a) $AB = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 8 \\ -1 & 4 \end{pmatrix}$

$|AB| = \begin{vmatrix} -1 & 8 \\ -1 & 4 \end{vmatrix} = -4-(-8) = 4 \neq 0 \quad \exists\, (AB)^{-1}$

$Adj(AB) = \begin{pmatrix} 4 & 1 \\ -8 & -1 \end{pmatrix} \rightarrow (Adj(AB))^t = \begin{pmatrix} 4 & -8 \\ 1 & -1 \end{pmatrix} \rightarrow (AB)^{-1} = \frac{1}{4}\begin{pmatrix} 4 & -8 \\ 1 & -1 \end{pmatrix}$

$$(AB)^{-1} = \begin{pmatrix} 1 & -2 \\ 1/4 & -1/4 \end{pmatrix}$$

b) $AB^t - A^tB = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix} =$

$= \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 8 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -9 \\ 3 & -3 \end{pmatrix}$

c) $B^tX + A^tB = A^t \rightarrow B^tX = A^t - A^tB \rightarrow (B^t)^{-1}B^tX = (B^t)^{-1}(A^t - A^tB)$

$\rightarrow X = (B^t)^{-1}(A^t - A^tB) = \begin{pmatrix} 1 & 1/2 \\ -1 & 0 \end{pmatrix}\left[\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -1 & 2 \end{pmatrix}\right] =$

$= \begin{pmatrix} 1 & 1/2 \\ -1 & 0 \end{pmatrix}\left[\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 8 \\ -1 & 4 \end{pmatrix}\right] = \begin{pmatrix} 1 & 1/2 \\ -1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 4 & -7 \\ 2 & -3 \end{pmatrix}$

$= \begin{pmatrix} 4+1 & -7-3/2 \\ -4+0 & 7+0 \end{pmatrix}$

$= \begin{pmatrix} 5 & -17/2 \\ -4 & 7 \end{pmatrix}$

$B^t = \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix} \rightarrow |B^t| = 2$

$Adj(B^t) = \begin{pmatrix} 2 & -2 \\ 1 & 0 \end{pmatrix} \rightarrow (Adj(B^t))^t = \begin{pmatrix} 2 & 1 \\ -2 & 0 \end{pmatrix} \rightarrow (B^t)^{-1} = \begin{pmatrix} 1 & 1/2 \\ -1 & 0 \end{pmatrix}$