MATEMÁTICAS CCSS II
ÁLGEBRA
PROBLEMA 24

JULIO 2015 B

Problema 1. Sean las matrices $A=\begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix}$, $B=\begin{pmatrix}2 & 2\\ 1 & -1\end{pmatrix}$ y $C=\begin{pmatrix}1 & -1\\ 1 & -3\end{pmatrix}$.

a) Halla la matriz X que satisface la ecuación $AX - BCX = 3C$.

b) Calcula la matriz inversa de $A^t + B$, donde A^t representa la matriz traspuesta de A.

a) $AX - BCX = 3C$

Sacamos factor común:

$(A - BC)X = 3C$

$(A-BC)^{-1}(A-BC)X = (A-BC)^{-1}\cdot(3C)$

$$\boxed{X = (A-BC)^{-1}\cdot(3C)}$$

Cálculo de $(A-BC)^{-1}$

$M = A-BC = \begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix} - \begin{pmatrix}2 & 2\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & -1\\ 1 & -3\end{pmatrix}$

$= \begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix} - \begin{pmatrix}2+2 & -2-6\\ 1-1 & -1+3\end{pmatrix}$

$= \begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix} - \begin{pmatrix}4 & -8\\ 0 & 2\end{pmatrix} = \begin{pmatrix}-3 & 10\\ -1 & 2\end{pmatrix}$

$|M| = -6+10 = 4 \neq 0 \quad \exists M^{-1}$

$Adj(M) = \begin{pmatrix}2 & 1\\ -10 & -3\end{pmatrix} \rightarrow (Adj(M))^t = \begin{pmatrix}2 & -10\\ 1 & -3\end{pmatrix}$

$M^{-1} = \frac{1}{4}\begin{pmatrix}2 & -10\\ 1 & -3\end{pmatrix}$

$$\Rightarrow X = \frac{1}{4}\begin{pmatrix}2 & -10\\ 1 & -3\end{pmatrix}\cdot\begin{pmatrix}3 & -3\\ 3 & -9\end{pmatrix} = \frac{1}{4}\begin{pmatrix}6-30 & -6+90\\ 3-9 & -3+27\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix}-24 & 84\\ -6 & 24\end{pmatrix} = \begin{pmatrix}-6 & 21\\ -3/2 & 6\end{pmatrix}$$

b) $A^t + B = \begin{pmatrix}1 & -1\\ 2 & 4\end{pmatrix} + \begin{pmatrix}2 & 2\\ 1 & -1\end{pmatrix} = \begin{pmatrix}3 & 1\\ 3 & 3\end{pmatrix} = N$

N^{-1}: $|N| = \begin{vmatrix}3 & 1\\ 3 & 3\end{vmatrix} = 9-3 = 6 \neq 0 \quad \exists N^{-1}$

$Adj(N) = \begin{pmatrix}3 & -3\\ -1 & 3\end{pmatrix} \qquad (Adj(N))^t = \begin{pmatrix}3 & -1\\ -3 & 3\end{pmatrix}$

$$N^{-1} = \frac{1}{6}\begin{pmatrix}3 & -1\\ -3 & 3\end{pmatrix} = \boxed{\begin{pmatrix}1/2 & -1/6\\ -1/2 & 1/2\end{pmatrix}}$$