MATEMÁTICAS CCSS II
ÁLGEBRA **JUNIO 2011 B**
PROBLEMA 6

Problema 1. Dadas las matrices:

$$A=\begin{pmatrix}1 & -2\\ -1 & 4\end{pmatrix},\quad B=\begin{pmatrix}1 & 0\\ -2 & -1\end{pmatrix}\quad \text{y}\quad C=\begin{pmatrix}3 & 1\\ 2 & -1\end{pmatrix}$$

a) Calcula la matriz inversa de la matriz *C*.

b) Obtén la matriz *X* que verifica $AX+B^t=C$, siendo B^t la matriz transpuesta de *B*.

a) $C=\begin{pmatrix}3 & 1\\ 2 & -1\end{pmatrix}$ $\quad |C|=-3-2=-5$

$$\text{Adj}(C)=\begin{pmatrix}-1 & -2\\ -1 & 3\end{pmatrix}\rightarrow (\text{Adj}(C))^t=\begin{pmatrix}-1 & -1\\ -2 & 3\end{pmatrix}\rightarrow C^{-1}=\frac{1}{-5}\begin{pmatrix}-1 & -1\\ -2 & 3\end{pmatrix}$$

$$\boxed{C^{-1}=\begin{pmatrix}1/5 & 1/5\\ 2/5 & -3/5\end{pmatrix}}$$

b) $AX+B^t=C$

$AX=C-B^t$

$A^{-1}\cdot AX=A^{-1}(C-B^t)$

$\boxed{X=A^{-1}(C-B^t)}$

Cálculo de A^{-1}:

$$|A|=\begin{vmatrix}1 & -2\\ -1 & 4\end{vmatrix}=4-2=2$$

$$(\text{Adj}(A))=\begin{pmatrix}4 & 1\\ 2 & 1\end{pmatrix}\quad (\text{Adj}(A))^t=\begin{pmatrix}4 & 2\\ 1 & 1\end{pmatrix}$$

$$A^{-1}=\frac{1}{2}\begin{pmatrix}4 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}2 & 1\\ 1/2 & 1/2\end{pmatrix}$$

$$\Rightarrow X=\begin{pmatrix}2 & 1\\ 1/2 & 1/2\end{pmatrix}\left[\begin{pmatrix}3 & 1\\ 2 & -1\end{pmatrix}-\begin{pmatrix}1 & -2\\ 0 & -1\end{pmatrix}\right]=$$

$$=\begin{pmatrix}2 & 1\\ 1/2 & 1/2\end{pmatrix}\begin{pmatrix}2 & 3\\ 2 & 0\end{pmatrix}=\begin{pmatrix}4+2 & 6+0\\ 1+1 & \frac{3}{2}+0\end{pmatrix}=\begin{pmatrix}6 & 6\\ 2 & 3/2\end{pmatrix}$$